Section 1.3

✓1.3.1 (c)

✓1.3.3 (b)

✓1.3.4

✓1.3.14 (a)-(e)

✓1.3.15 (a)-(d)

✓1.3.20 (a), (b), (e)

✓★ 1.3.22 (c), (e)

(1.3.25)

✓1.3.26

✓1.3.29

(1.3.32(a))

(1.4.2 (a)+(b))

✓ 1.4.4

✓★ 1.4.5. (a)

✓1.4.6.

1.3.1 (c)

Solve $\begin{aligned} x - 2y + z &= 0 \\ 2y - 8z &= 8 \\ -4x + 5y + 9z &= -9 \end{aligned}$ using an augmented matrix.

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right) \xrightarrow{4R_1 + R_3} \left(\begin{array}{cccc} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{array}\right)$$

$$\xrightarrow{3/2 R_2 + R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right) \rightarrow \boxed{z = 3} \Rightarrow 2y - 8(3) = 8 \Rightarrow 2y = 32 \Rightarrow \boxed{y = 16}$$

$$x - 2(16) + 3 = 0$$

$$\Rightarrow x - 29 = 0$$

$$\Rightarrow \boxed{x = 29}$$

check $-4(29) + 5(16) + 9(3) = -9$. ✓

1.3.3 (b) Solve by Gaussian Elimination:

$$\begin{pmatrix} 6 & 1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 6 & 1 & 5 \\ 3 & -2 & 5 \end{array}\right) \xrightarrow{-1/2 R_1 + R_2} \left(\begin{array}{cc|c} 6 & 1 & 5 \\ 0 & -2.5 & 2.5 \end{array}\right) \rightarrow -2.5v = 2.5 \Rightarrow \boxed{v = -1}$$

$$6u + 1(-1) = 5$$

$$\Rightarrow 6u = 6$$

$$\Rightarrow \boxed{u = 1}$$

check $3(1) - 2(-1) = 5$. ✓

1.3.4 Find the equation of the parabola $y = ax^2 + bx + c$ that goes through the points $(1,6), (2,4), (3,0)$

$$a(1^2) + b(1) + c = 6$$
$$a(2^2) + b(2) + c = 4$$
$$a(3^2) + b(3) + c = 0$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 0 \end{array}\right) \xrightarrow[-9R_1 + R_3]{-4R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -3 & -20 \\ 0 & -6 & -8 & -54 \end{array}\right)$$

$$\xrightarrow{-3R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -3 & -20 \\ 0 & 0 & 1 & 6 \end{array}\right) \rightarrow \boxed{c = 6}$$

$$-2b - 3(6) = -20$$
$$\Rightarrow -2b - 18 = -20$$
$$\Rightarrow -2b = -2$$
$$\Rightarrow \boxed{b = 1}$$

$$a + 6 + 1 = 6$$
$$\Rightarrow \boxed{a = -1}$$

$$\boxed{y = -x^2 + x + 6}.$$

1.3.14 (see bk pg 19)

(a) $-2R_2 + R_1$ for $2 \times n$ matrix

(b) $7R_1 + R_2$ for $2 \times n$ matrix

(c) $-5R_3 + R_2$ for $3 \times n$

(d) $\frac{1}{2}R_1 + R_3$ for $3 \times n$

(e) $-3R_4 + R_2$ for $4 \times n$

1.3.15

(a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

1.3.20 write down explicit requirements on its entries a_{ij} s.t a square matrix A be

(a) diagonal

$a_{ij} = 0$ for $i \neq j$

(b) upper triangular

$a_{ij} = 0$ for $i > j$

(e) special lower triangular

$a_{ij} = 0$ for $i < j$

and $a_{ii} = 0$ for all i.

★ 1.3.22

Find the LU factorization of

(c) $\begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix} \xrightarrow[-R1+R3]{R1+R2} \begin{pmatrix} -1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = U$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

neg of weights

Check: $LU = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 3 \end{pmatrix} \checkmark$.

(e) $\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 3 & 2 \end{pmatrix} \xrightarrow[R_1+R_3]{2R_1+R_2} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 3 & 2 \end{pmatrix} \xrightarrow{R_2+R_3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{pmatrix} = U$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$.

Check: $LU = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 3 & 2 \end{pmatrix} \checkmark$.

1.3.26 True or false: If A has a zero entry on its main diagonal, it is not regular.

False: E.g. $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} \xrightarrow{R_1 + R_2} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ A is regular.

1.3.29 Prove that the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ does not have an LU factorization.

Proof.

If $A = LU = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \begin{pmatrix} b & c \\ 0 & d \end{pmatrix} = \begin{pmatrix} b & c \\ ab & ac+d \end{pmatrix}$
$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Then $b = 0$
$ab = 1$. this is not possible.

So A cannot have an LU factorization.

save 20 min for

1.4.5 (a) Show that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is nonsingular iff $ad - bc \neq 0$

$\boxed{\Rightarrow}$ Assume A is nonsingular.

If $a \neq 0$, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{-c/a\, R_1 + R_2} \begin{pmatrix} a & b \\ 0 & -\frac{bc}{a} + d \end{pmatrix}$.

And since A is nonsingular, the pivots are not zero, so $-\frac{bc}{a} + d \neq 0$.

$\Rightarrow -\frac{bc}{a} + \frac{ad}{a} \neq 0$

$\Rightarrow \frac{ad - bc}{a} \neq 0$

$\Rightarrow ad - bc \neq 0$.

If $a = 0$, then swap R_1 and R_2:

$$\begin{pmatrix} 0 & b \\ c & d \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} c & d \\ 0 & b \end{pmatrix}.$$

Since A is nonsingular, the pivots cannot be zero, so $c \neq 0$, $b \neq 0$

$$\Rightarrow bc \neq 0$$

Thus $ad - bc = 0d - bc = -bc \neq 0$.

$\boxed{\Leftarrow}$ Assume $ad - bc \neq 0$.

If $a \neq 0$, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{-\frac{c}{a}R_1 + R_2} \begin{pmatrix} a & b \\ 0 & \frac{-bc}{a} + d \end{pmatrix}$

And $\frac{-bc}{a} + d = \frac{-bc}{a} + \frac{ad}{a} = \frac{ad - bc}{a}$,

and since $ad - bc \neq 0$, $\frac{ad-bc}{a} \neq 0$,

Thus the pivots are not zero,

so A is nonsingular.

If $a = 0$, then $ad - bc = -bc \neq 0$

$\Rightarrow b \neq 0$ and $c \neq 0$

So $\begin{pmatrix} 0 & b \\ c & d \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} c & d \\ 0 & b \end{pmatrix}$

Pivots $\neq 0$. So A is nonsingular.

1.4.4 Find the equation $z = ax + by + c$ for the plane passing through $p_1 = (0, 2, -1)$, $p_2 = (-2, 4, 3)$ and $p_3 = (2, -1, -3)$

$$2b + c = -1$$
$$-2a + 4b + c = 3$$
$$2a - b + c = -3$$

$$\left(\begin{array}{ccc|c} 0 & 2 & 1 & -1 \\ -2 & 4 & 1 & 3 \\ 2 & -1 & 1 & -3 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} -2 & 4 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 2 & -1 & 1 & -3 \end{pmatrix}$$

$$\xrightarrow{R_1 + R_3} \begin{pmatrix} -2 & 4 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 3 & 2 & 0 \end{pmatrix} \xrightarrow{-3/2 R_2 + R_3} \begin{pmatrix} -2 & 4 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 0.5 & 1.5 \end{pmatrix}$$

$\Rightarrow \quad 0.5c = 1.5$

$\Rightarrow \quad \boxed{c = 3}$

$2b + 3 = -1$

$\Rightarrow \quad 2b = -4$

$\Rightarrow \quad \boxed{b = -2}$

$-2a + 4(-2) + 1(3) = 3$

$\Rightarrow \quad -2a - 5 = 3$

$\Rightarrow \quad -2a = 8$

$\Rightarrow \quad \boxed{a = -4}$

$$\boxed{y = -4x - 2y + 3}$$

1.4.6 True or False: A singular matrix cannot be regular.

True: Every reg. matrix is nonsingular (contrapositive).